Problem-based Learning in Mathematics: How Teacher Actions and Beliefs Impact the Level of Cognitive Demand

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PROBLEM-BASED LEARNING IN MATHEMATICS: HOW TEACHER ACTIONS AND BELIEFS IMPACT THE LEVEL OF COGNITIVE DEMAND

Thesis by
Megan Pacheco

ABSTRACT

Purpose of the Study:
Problem-based learning is a student-centered approach to teaching mathematics that helps students to develop critical thinking and problem-solving skills while learning the mathematics content. The purpose of this study is to explore how teacher actions and beliefs impact the level of cognitive demand in a problem-based setting.

Procedure:
In order to examine how teacher actions and beliefs impact the level of cognitive demand of a problem, this study compares the implementation of a high level problem by two secondary mathematics teachers. Each teacher's implementation of the problem was observed, recorded, and analyzed in order to find evidence of factors that have been found to maintain a high level cognitive demand. A pre- and post-implementation survey was also administered in order to better understand the beliefs and actions of the teacher.

Findings:
Analysis of the implementation of the problem revealed that both teachers struggled to consistently maintain a high level of cognitive demand throughout the lesson, although both teachers exhibited strengths and challenges. The teacher who demonstrated more evidence of the factors associated with high levels of cognitive demand had beliefs and goals for student learning that were more aligned with a problem-based approach.

Conclusions:
Both teachers had mixed results in terms of maintaining a high level of cognitive demand. When implementing a problem-based approach, teachers need opportunities to collaborate and receive feedback on their practice in order to gain a better understanding of the strategies for maintaining a high level of cognitive demand. Additional research is also needed to understand how teacher beliefs are formed and how those beliefs impact student performance.

Chair: Dr. Katherine Morris

Signature: ___________________________ Date: 5/2/12
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CHAPTER 1
INTRODUCTION

As a mathematics educator, both in the classroom and supporting other math teachers outside of the classroom, I have become passionate about creating learning opportunities for students that are engaging and push students to deeper learning. I believe mathematics education should surpass basic knowledge of key skills and concepts and help students to develop the problem-solving and reasoning skills that they can apply in any context. The study of mathematics provides an ideal context for investigating ideas, analyzing data, making sense of information, developing logical reasoning skills, and constructing viable arguments. In addition, communication is a critical aspect of the learning process, as it provides a means for the sharing, critique, and extension of ideas (Cazden, 2001).

Sadly, many of these aspects are lacking in math classrooms today. Pressure to perform on standardized tests, cover a large amount of material, and conform to long standing notions of what mathematics education should look like has left many math teachers feeling boxed into a certain instructional methodology that focuses on the acquisition of basic skills and fluency with those skills. This type of methodology is typically characterized by direct instruction that is focused on definitions and procedures, followed by students mimicking the work of the teacher through repeated practice of the demonstrated procedures (Stigler & Hiebert, 1999).

There are other teaching strategies that are argued for in the educational literature and used in countries that typically perform higher than the US on international comparisons (Stigler & Hiebert, 1999; Stevanson & Stigler, 1994). American teachers have little exposure to such variations in instructional methods. More exposure to
instructional strategies that challenge the idea that math is merely a set of routines and procedures would assist math teachers in transitioning their instructional practices beyond the lecture-based strategy that is so common today.

During my time in the classroom, I found great success with instructional strategies that were more collaborative and inclusive of students in the problem-solving process. In particular, I utilized a project- and problem-based approach, in which the students were given a complex task that required them to investigate ideas, apply previous knowledge, collect and analyze data, communicate their ideas, and make validated conclusions. When the task was well constructed, my students were engaged and challenged. They developed problem-solving skills and began to see the value of mathematics beyond the confines of the classroom. As I transitioned to an instructional coach, I wanted to be able to support other math teachers in this approach. In particular, I was interested in what challenges teachers might encounter and what support I could provide in order to assist them in becoming more adept at utilizing a problem-based approach. I quickly observed that many teachers struggled with the creation or selection of quality tasks, as well as the level and type of facilitation necessary to support student learning in this methodology. I also observed that even when teachers used a rich and challenging task, the students’ responses to that task often resulted in the teacher lowering the level of the cognitive demand for their students. For example, when their students were struggling, many teachers would show the students how to do the problem or give them an explanation rather than using strategies that would help students to make sense of the information on their own. It was those observations that initiated my interest
in better understanding teacher actions and how they shape the learning experience for students.

At the time of this study, I had been working with a group of math teachers on the use of problem-based learning. This group of teachers participated in a series of professional development workshops on the use of problems. The workshops focused on understanding the qualities of a well designed problem, facilitation strategies for a problem-based setting, and analyzing case studies of classrooms that utilized a problem-based approach. In addition, these teachers were part of a larger group of math teachers that were piloting a set of well-established mathematical tasks specifically designed to promote deeper mathematical thinking and problem-solving skills. This presented a unique opportunity to study the nature of teachers' implementation of cognitively demanding problems with their students and the multitude of instructional decisions and actions that teachers inevitably make in the process of guiding student learning. It also provided an opportunity to explore how the beliefs and prior experiences of the teachers may have contributed to their implementation of a problem-based approach.

Problem Statement

This study is a qualitative analysis of teachers' implementation of a problem-based approach in their classroom. It explores how two teachers with different beliefs about teaching and learning in mathematics, different prior educational experiences, and different instructional styles implement a problem with a high level of cognitive demand. The teacher in each case participated in a series of professional development sessions I conducted that focused on the use of problem-based learning. The central question of
this study is "how do teacher actions and beliefs impact the level of cognitive demand in a problem-based setting?"

Significance of the Study

There is a growing awareness that mathematics education needs to find a greater balance between helping students to learn basic skills and procedures and developing a deeper conceptual understanding of the content. Student-centered approaches such as problem-based learning allow for students to actively engage in the learning process rather than passively repeating the skills and procedures demonstrated by the teacher. As more and more teachers explore and embrace this type of instructional approach, a better understanding of the instructional challenges associated with problem-based learning will be needed in order to support teachers.

In conducting a study of the teachers' implementation of a high level demand problem, I wanted to explore the complexities of teaching using a problem-based approach and the teacher actions that maintain or lower the level of the cognitive demand. This not only informs my work with these specific teachers, but also the broad range of teachers that I work with as an instructional coach and has implications for other professional development of mathematics teachers at large.

An examination of how these teachers approach and implement the problem will provide a greater sense of the needs of the teachers and the type of support required to help them to effectively create engaging and rigorous learning experiences for their students. In addition, this study can help to inform other teachers and administrators who are considering using a problem-based approach to teach mathematics.
Furthermore, this study will elicit new questions about teaching and learning in mathematics and the teacher actions and beliefs necessary to promote deeper learning in students. These questions may serve to inform additional research that is needed in the field of mathematics education. Overall, my hope is that this study will contribute to the growing body of research and awareness around the shift to a more problem-based approach that is both needed in mathematics education and entailed in the implementation throughout the United States in the new Common Core Standards for Mathematics, as well as the viability of that shift.

**Definition of Terms**

This study explores the use of problem-based learning as a means for maintaining a high level of cognitive demand for student learning. Definitions of the key terms associated with that practice are as follows:

- **Classroom discourse.** The dialogue used to exchange and communicate ideas between and among student(s) and teacher(s) during the learning process.

- **Cognitive demand.** The depth of knowledge required by a task or assessment as related to the level of reasoning, self-monitoring, connections, and sense making required to produce a response.

- **Didactic Teaching.** Instruction that emphasizes the transfer of knowledge from teacher to student through explanation, repetition, and examples.

- **Inquiry-based learning.** A process by which learners construct understanding by asking questions and investigating ideas through active exploration.
• **Problem-based learning.** A student-centered instructional strategy in which students collaboratively solve problems through in-depth investigations of the concepts and skills.

• **Scaffolding.** The support provided to students while learning a new skill or concept.

**Limitations**

The purpose of this study is to explore the teacher actions that lead to the maintenance or decline in cognitive demand for student learning. By examining the instructional practices and decisions of two teachers with different backgrounds and beliefs on teaching and learning in mathematics, the study will expose potential implications and areas for further research. However, it will not completely address many of the variables that effect student success in the classroom. The research will not consider the students’ backgrounds and prior experiences that may play a role in their response to the problem and subsequent teacher actions, such as any prior exposure to a problem-based approach. Nor was data collected about the students’ performance in the class or on standardized tests. It is reasonable to assume that students that have had experience working in a classroom that utilized a problem-based approach would respond more positively to the mathematical task given in this study and therefore would potentially be able to operate within a higher level of cognitive demand.

In addition, although both teachers have relatively the same amount of exposure to problem-based learning, the study does not examine their complete teacher preparation, professional development, or classroom experiences that inevitably influence their instructional practices and beliefs. Teachers that have been exposed to and have had
the opportunity to practice instructional strategies that are aligned with the goals and
theory behind a problem-based approach may find it easier to implement the task and
focus on strategies for maintaining a high level of cognitive demand. For example,
inquiry-based instructional practices or strategies that invoke a level of discourse among
students would reinforce a teacher's ability to engage students in the mathematics at a
deeper level. A teacher with limited prior exposure to these practices may struggle with
the implementation of the task, despite their own beliefs about teaching and learning in
mathematics.

Another consideration for the study is that the research is relatively small in
scope. Each teacher was observed implementing one task for one class period, which is
merely a snapshot of their classroom in the course of a school year. As such, factors
outside the control of the classroom teacher affect the tone and productivity of a
classroom may be evident or even prominent in the findings in ways they wouldn't be in
a longer study. These factors may include outside disruptions, the students' state of
mind, and changes to the school day. Thus there is no way to ensure that the lesson
observed depicts a typical classroom experience for either the teacher or their students.

While this study will begin to make inferences into the work of teachers
implementing a problem-based approach in mathematics, it cannot make definitive
conclusions due to the many complexities of teaching and learning. Furthermore, while
there are many ways in which these two teachers are typical with respect to their year of
experience, day-to-day practice, etc., the study is limited in the extent to which the
findings can be generalized to the broader teaching community of practice. Therefore, the
goals of the study are to highlight specific commonalities and differences between the
two teachers in order to inform considerations for ongoing support and training of math teachers, as well as potential areas for more in-depth research. These considerations may point to areas that would meet a wide range of teachers seeking to improve their practice.
CHAPTER 2

LITERATURE REVIEW

Mathematics education in the United States has a long history of using primarily lecture-based instructional approaches that focus heavily on the procedural aspect of mathematics and memorization of basic skills and facts (Stigler & Hiebert, 1997; Tarmizi, Tarmizi, Lojinin, & Mokhtar, 2010). Analysis of mathematics classrooms across the nation indicates that a majority of classroom instruction is composed of students passively listening to explanations and then mimicking the work of the teacher or worked out examples, which leads to limited content knowledge (Cotic & Zuljan, 2009; Mevarech & Kramarski, 2003; Stigler et al., 1997). For many decades now, national organizations, such as the National Council of Teachers of Mathematics (NCTM), have long argued for a shift away from instruction that emphasizes basic procedural knowledge to instructional practices that engage students in deeper conceptual understanding of mathematics in order to make connections between ideas and to develop reasoning skills (National Council of Teachers of Mathematics, 2000).

Research indicates that the types of tasks that students engage with determine the depth of student understanding of the mathematical ideas (Stein, Grover, & Henningsen, 1996). NCTM (2000) recommends a shift away from memorization and rote application of procedures towards standards of performance based on conceptual understanding and reasoning. This need for a change in instruction aligns with the use of a problem-based learning, which is characterized by many of the attributes outlined by NCTM and an increasing body of research. This literature review addresses how problem-based learning
in mathematics can increase cognitive demand of student learning, as well as implications on teachers' beliefs and instructional strategies.

Why Use a Problem-based Approach?

Overview of Problem-based Learning

When a student is presented with a problem in which they are already familiar with the strategy or ideas for solving the problem, the problem is merely an exercise in practicing procedures (Cotic et al., 2009, Mevarech et al., 2003). However, if the problem is altered or designed in a manner such that there is no clear path for solving the problem and students must use and extend prior knowledge in a novel way, then the learning becomes deeper and more complex. That is to say it becomes authentically problematic and worthy of solving. Students need opportunities to discover for themselves ways to tackle problems and what information is critical for solving the problem at hand. In addition, it is important that students have an opportunity to collaborate and discuss mathematical ideas with their peers in order to extend ideas and make connections between various mathematical concepts (Cerezo, 2004; Cotic et al., 2009; Mevarech et al., 2003; Tarmizi et al., 2010). As opposed to presenting students with problems after they have learned the skills, problem-based learning encourages knowledge construction by starting each learning experience by presenting students with a complex problem (Cotic et al., 2009; Tarmizi et al., 2010). In order to solve the problem, students must apply prior knowledge, ask questions, and think critically about both what a reasonable solution might entail and what strategy would be best for tackling the problem.
Problems in this setting are often considered open-ended problem in which students, often collaboratively, may develop varied strategies for approaching and solving the problem (McDuffie & Mather, 2006). Thus the sharing out and discussion of ideas, or classroom discourse, is a critical aspect of problem-based learning. Essentially, the emphasis in problem-based learning is not on the answer, but rather the process of solving the problem (Cerezo, 2004; Tarmizi et al., 2010). Most often the process involves working in small groups, an initial exploration of ideas, refining and crafting a solution, presentation of work, and final reflection on the process and key ideas (Cerezo, 2004).

Through the use of problems, teachers and students engage in inquiry, discourse, and shared construction of knowledge. The role of the teacher shifts to facilitator, and the students become both problem-solvers and self-evaluators. As a result, students have an opportunity to learn more than just limited procedural knowledge that is typically covered in a more conventional approach.

**Strengths and Challenges of Problem-based Learning**

While quantitative research on the use of a problem-based approach in mathematics at the secondary level is somewhat limited, several recent studies have indicated positive results with the use of problems. In a classroom-based study conducted by Tarmizi et al. (2009) a group of students that were exposed to a problem-based approach were compared to a control group that used conventional methods. While there was no significant difference in the mean scores of the overall mathematics performance between the PBL group and the control group on a post test, the problem-based group did demonstrate a difference in the amount of effort that students put forth to
solve problems during both the learning phase and post test. The researchers measured the amount of mental effort using the Paas Mental Effort Rating Scale (PMERS), a scale that assesses various indicators, such as ability to remain self-directed, asking insightful questions, showing interest in learning, and having a positive attitude. However, this particular study used a small sample size and students had limited exposure to the use of problems. In a more comprehensive study that took place over the course of a year, students in an experimental group that were taught using a problem-based approach were compared to a control group that was taught using traditional methods that primarily focused on learning how to master algorithms (Cotic et al., 2009). The students in the experimental group were more successful in solving more difficult problems than the students in the control group. However, there was no significant difference in the students’ ability on computing basic operations.

The findings in Cotic et al.’s (2009) study are also in agreement with the results obtained by other research studies on the use of problem-based learning (Cerezo, 2004; Mevarech et al., 2003; Wildmon, Skinner, McCurdy & Sims, 1999). In these studies, students involved in problem-based learning classrooms showed a better understanding of key concepts and a greater ability to apply the strategies they had learned through the problem-solving process. In addition, students in these studies reported a positive association with problems and learning mathematics and the data revealed that students believed that the use of problems helped them to be more confident and independent learners. According to Cerezo (2004), the students in a problem-based learning classroom demonstrated increased self-efficacy and interest in the topic and the students
perceived that problem-based learning helped them learn more and contributed to their overall motivation and success.

Wildmon et al. (1999) found that by increasing the number of problems in the curriculum, students were actually more engaged and opted for assignments and activities that used more complex problems over basic computational problems or simple word problems. Similarly, Mevarech et al. (2003) found that students who received metacognitive training, or a clear process for problem-solving, found more success in cognition and motivation with a problem-based approach. In addition, they also demonstrated more persistence in the problem-solving process. This was a common finding in studies where students were exposed to problem-based strategies over significant amount of time (Cerezo, 2004; Goos, 2004; Mevarech et al., 2003).

Despite the positive results associated with problem-based learning, this approach requires a dramatic shift in both teacher practices and beliefs (Rickard, 2005; Sherin, 2002; Spronken-Smith & Harland, 2009). Research indicates that student learning is not solely determined by the instructional strategies and materials, but also the teacher’s ideas about student learning, their content knowledge, and professional development experiences (McDuffie et al., 2006). According to Rickard (2005), even teachers who expressed support for problem-based learning still spent substantial amount of instructional time teaching rote algorithms and having students practice computational exercises. In addition, a problem-based approach requires deeper content knowledge so that teachers can respond to the various student approaches and ideas in ways that extends student ideas and guides them to deeper content knowledge (Goos, 2004; Spronken-Smith et al., 2009). Spronken-Smith et al. (2009) found that teachers
experienced a wide variety of problems transitioning to a problem-based approach and were somewhat resistant to move away from their previous instructional strategies, even when supported by professional development and seeing positive results in their classroom with a problem-based approach.

Although the use of problems does not necessarily increase students’ computations skills and teachers struggle with the adjustment of instructional strategies, problem-based learning can offer a variety of noteworthy positive results in student learning. These strengths include the type of skills that will serve students throughout their mathematics education and beyond, including problem-solving skills, persistence, increased engagement, and deeper understanding of the mathematical concepts.

**Using Problems to Increase Cognitive Demand**

**Defining Cognitive Demand**

While research shows that problem-based learning can increase students’ conceptual understanding, the type of problem selected for students is critical to the level of student learning (Arbaugh & Brown, 2005; Boston & Smith, 2009; Henningsen & Stein, 1997; Stein, Grover, & Henningsen, 1996). Stein et al. (1997) found that the type of mathematical task that students complete greatly affects the students’ ability to learn to think mathematically. For example, tasks that simply ask students to replicate a procedure result in very low level mathematical thinking while tasks that force students to make connections and apply previously learned skills to new and complex situations lead to higher levels of mathematical understanding and generalization of ideas. As such, Stein, Smith, Henningsen, and Silver (2000) developed a Task Analysis Guide (see Appendix D) which characterized problems into four distinct categories, ranging in level of
cognitive demand, or mental skills and abilities. The lowest level of problems are memorization problems, in which students are asked to reproduce or commit to memory previously learned facts, rules, formulae, or definitions. The highest level of problems is categorized as doing mathematics, which requires complex non-algorithmic thinking and exploration and understanding of the concepts and relationships between those concepts.

**Instructional Strategies Associated with High Levels of Cognitive Demand**

By definition high level cognitive demand problems are much more ambiguous and require more personal risk on the part of students than low level problems (Henningsen et al., 1997). As such, the way in which a problem is scaffolded, or structured for student learning, becomes a critical component of a problem-based approach (Arbaugh et al., 2005; Goos, 2004; Henningsen et al., 1997; Stein et al., 1996). This scaffolding includes the types of questions teachers ask, inviting classroom discourse, creating a classroom of inquiry, and interweaving of spontaneous and theoretical concepts. White (2003) found that teachers who maintained a high level of cognitive demand utilized four general questioning patterns, including valuing students' ideas, exploring students' answers, incorporating students' background knowledge, and encouraging student-to-student communication. These questioning patterns supported student thinking and communication of ideas in ways that led to deeper understanding of the content.

A high level of cognitive demand was also apparent in classrooms that placed a significant emphasis on discourse, or the sharing of ideas between students. In the context of problem-based learning, mathematical discourse involves not only the sharing of ideas, but also building a classroom environment in which students feel comfortable
questioning and expanding on the ideas of others (Goos, 2004; Sherin, 2002). This level of interaction between peers results in the co-construction of new knowledge and opportunities to make sense of a variety of strategies. Goos (2004) found that by establishing classroom norms and practices that supported classroom discourse, students were able to tackle problems with a higher level of cognitive demand. However, studies have also shown that teachers struggle with facilitating mathematical discourse and finding the balance between allowing students to explore ideas and keeping the discussion mathematically productive (Blanton & Kaput, 2005; Breyfogle, 2005; Sherin, 2002; White, 2003). Sherin (2002) found that by using a filtering approach, teachers are able to maintain a focus on the key mathematical ideas of the problem. This filtering approach includes allowing students to share out, highlighting the ideas that are most closely aligned with the concepts, and encouraging deeper exploration of those ideas. However, techniques such as filtering rely heavily on teachers' ability to listen to student ideas and explanations of the mathematical concepts so that they can respond to and guide student thinking in a constructive manner that maintains a high level of cognitive demand (Davis, 1997).

Many of these findings support the work of Henningsen et al. (1997), which concluded that the factors that have the greatest impact on student engagement and learning include building on prior knowledge, scaffolding of tasks, providing the appropriate amount of time, modeling high level performance and sustained pressure for explanation and meaning. Various factors associated with a decline of cognitive demand include providing too much or too little time, shifting the focus from problem-solving process to correct answer, inappropriateness of tasks, and classroom management issues.
As such, teachers need support and training in how to design cognitively demanding problems as well as how to provide a classroom experience in which students can engage with the task in a way that leads to deeper learning.

**Implications on Teacher Beliefs, Practices, and Professional Development**

**The Role and Beliefs of Teachers**

A change in instructional strategies also requires a shift in role of the teacher to a facilitator rather than content expert (Doerr, 2006). Sponken-Smith et al. (2009) found that newer teachers struggled more than experienced teachers with this shift and that this resulted in inequities with how the teacher responded to and supported student learning. Blanton et al. (2005) conducted a case study of a teacher over the course of a school year, and found that through sustained practice in the use of problems, the teacher was able to adjust their role, instructional strategies, and beliefs in order to focus their support on the problem-solving process and understanding of concepts. In addition, Doerr (2006), who worked extensively with a group of teachers, found that by shifting their role to facilitator the teachers were able to develop a more sophisticated schema for understanding the diversity and depth of student thinking and was therefore better able to support student engagement with the task. The teachers guided students to refine and revise their own work, which led to more connections and greater depth of understanding. Swan (2005) also found that teachers who viewed mathematics as an interconnected body of ideas and reasoning process were better able to sustain effective instructional practices aligned with a problem-based approach. In this view, learning is believed to be a collaborative effort between students and the teacher where concepts are explored through discussion and inquiry in order to gain a deeper conceptual understanding.
Overall, research has shown that a combination of shifts in beliefs, practices, and role is necessary to sustain a problem-based approach that evokes higher levels of cognitive demand for student learning. Through professional development, including both formal training on the use of problems, and informal opportunities such as communities of practice, teachers have demonstrated the ability to make the necessary shifts.

Professional Development for Teachers

Various professional development approaches have been utilized and studied in order to determine what type of approach leads to sustained change in teachers’ beliefs about the teaching and learning of mathematics and the instructional strategies they utilize. The use of problem-based learning requires a shift that is not always easy for teachers and often requires ongoing support and training (Rickard, 2005). Arbaugh et al. (2005) found that teachers that participated in professional development in which they examined the quality of problems tended to select problems with a higher level of cognitive demand with a higher frequency. In addition, several research studies have shown that professional development should include opportunities for teachers to revise and refine their own instruction through reflection and dialogue with colleagues as they try out new instructional strategies (Breyfogle, 2005; Davis, 1997; Rickard, 2005;). This type of professional development is often a component of communities of practice (COP) in which teachers engage in collaborative discussions and examination of both instructional practices and student work. Breyfogle (2005) found that through continual reflection, teachers were able to move through various levels of reflection on their instructional practices, starting with explaining and evolving to exploring. Teachers that
reached the exploring state were better able to implement a problem-based approach that met higher levels of cognitive demand for student learning.

Another study of teacher professional development around the use of problems, found that examining student work led to deeper insights into the types of problems and instructional strategies that were most effective for student learning (Doerr, 2006). In essence, examination of one’s own practices through careful consideration of problems, reflection on student responses, and dialogue with colleagues has a substantial influence on a teachers’ ability to effectively implement problem-based learning (Arbaugh et al., 2005; Breyfogle, 2005; Boston et al., 2009; Davis, 1997; Doerr, 2006; Rickard, 2005).

In addition to the type and opportunities for professional development, the type of instructional materials teachers are exposed to affected their instructional strategies (Arbaugh et al., 2005; McDuffie et al., 2006). In a study conducted by McDuffie et al. (2006) a teacher that was trained on the use of problem-based instructional materials shifted her views on teaching and learning as she adopted more problem-based practices. During this shift, the teacher began to view instructional materials as tools to meet students’ needs and was therefore able to focus more on student engagement with those materials. Other studies have found that teachers go through various stages of adjusting their use of instructional materials, including transforming, adapting, and negotiating (Sherin, 2002). While each stage showed evidence of the materials aiding in the development of the teacher’s own content knowledge, more advanced stages included the teachers using their new content knowledge in order to make changes to the lesson as it unfolds in ways that improved student learning.
Summary

The studies referenced in this literature review have indicated that problem-based learning can provide increased opportunities for students to engage at a deeper level with the mathematical content. Well designed problems require students to connect ideas, formulate new concepts, and apply previously learned skills to abstract situations. However, in order for problem-based learning to be effective and invoke these higher levels of cognitive demand, both students’ and teachers’ perceptions about the nature of learning mathematics must adjust to align with the instructional strategies found most effective for this approach. Teachers ought be able to design or select problems with a high level of cognitive demand and then create and facilitate a classroom environment in which students are routinely expected to engage in inquiry, problem-solving, and discourse. As evidenced by the research studies on the use of problems and associated professional development, teachers need support in both the development and implementation of problems to maintain high levels of cognitive demand. Regular and focused opportunities for teachers to engage in collaborative dialogue, reflect on their own practices, and refine their ability to respond to students’ needs is critical for the successful implementation of a problem-based approach.

Due to the fact that research in this area is somewhat limited, additional research is needed on the effectiveness of a problem-based approach, specifically at the secondary level, as a means for engaging students in deeper levels of understanding. Increased research in this area could help to inform more teachers and schools and lead to the type of change the NCTM and other organizations in the field of mathematics have been arguing for over the last several decades. In addition, access to professional development
around the use of problems in math is not yet made routinely made available for teachers, and therefore many math teachers continue to rely on conventional practices that result in one-dimensional learning of basic skills and procedures, as opposed to a dynamic set of practices that allow more students to engage with the content at a deeper level. Increasing access to professional development will help to equip teachers with skills and knowledge necessarily to effectively engage their students in rigorous, multi-dimensional learning experiences.
CHAPTER 3

METHODOLOGY

This study is intended to provide additional research on the use of problem-based learning in mathematics for increasing the level of cognitive demand of student learning. Previous research in this area indicates that both the quality of the problem and the teacher actions greatly impact the level of student success. While a problem with a high level of cognitive demand can provide an opportunity for meaningful learning to occur, teacher beliefs and corresponding actions ultimately decide the type of learning that results from a lesson. As shown in the Mathematical Tasks Framework (Stein & Smith, 1998, p. 4) in Figure 1, all tasks undergo a filtering process that begins with the initial design of the task, proceeds through the set-up of the task, and culminates with the implementation of the task. The student learning is a result of how each of these factors interacts with each other.

![Diagram](Figure1. Framework for the implementation of mathematical tasks.)

In order to focus on and understand how teacher actions and beliefs affect student learning, two teachers with similar exposure and training in problem-based learning were selected for this study in order compare their implementation of the same mathematical task and assess the level of cognitive demand throughout each lesson. Although both
teachers were open to the use of problem-based learning, the teachers represented a range in beliefs about teaching and learning in mathematics. This study explores their implementation of the problem and the actions that led to the maintenance or decline of the level of cognitive demand for their students.

Selection of the Participants

For the purpose of this study, I selected two secondary mathematics teachers that represented the range of teachers that I work with as an instructional coach. By studying their implementation of a common problem, I hoped to gain insight into how teachers approach the use of problems with their students and any success and challenges they may encounter. In turn, this will allow me to consider appropriate supports and professional development for teachers implementing a problem-based approach in their classroom.

In addition to representing the types of teachers that I work with, these teachers were selected based on the fact that they had relatively equivalent prior experience with using problem-based learning with their students. Both teachers had attended a series of professional development sessions on the use of problems in mathematics and had implemented several problems in their classrooms. The professional development sessions occurred over the course of a year and focused on the selection and design of cognitively demanding problems, as well as facilitation strategies for implementing the problems with students. In addition, both teachers taught the same course, high school geometry, in the same school district. They therefore followed the same pacing guide and had the same textbooks and curriculum resources available to them. Both teachers described their student population as representative of the district demographics, which is
approximately 50% Hispanic, 34% Caucasian, 6% Filipino, 4% multi-racial, 3% African American/Black, and 22% English Learners. When asked about the skill level of their students, both teachers felt that overall their students were relatively low skilled, even though their scores on standardized tests rated them as mostly proficient and above. From prior experience of working with these teachers, I knew that the teachers had somewhat of a different instructional style and beliefs about the nature of teaching and learning mathematics.

Selection of Task

Research shows that the selection of the task has a significant impact on “students’ opportunities to learn and on their perceptions about what mathematics is (Lappan & Briars, 1995).” Similarly, the type of instructional materials teachers are exposed to affected their instructional strategies (Arbaugh et al., 2005; McDuffie et al., 2006). Thus the selection of the task was a critical component of this study. As described in the Mathematical Tasks Framework shown in Figure 1, the problem selection is the first filter that determines what students learn. As stated by Stein and Lane (1996), “if we want students to develop the capacity to think, reason, and problem solve then we need to start with high level, cognitively complex tasks.” Therefore, the selection of the task was an important part of the process for ensuring students had an opportunity to engage in deep mathematical thinking.

The problem used in this study is part of a collection of tasks developed by the Mathematics Assessment Project (MAP), a collaborative project between the University of California, Berkeley, the Shell Center at the University of Nottingham, and the Silicon Valley Mathematics Initiative. These tasks are aligned to the Common Core Standards
(CCSS) and are intended to “both reveal and develop students’ understanding of key mathematical ideas and applications.” In addition, the tasks are designed to promote classroom discourse and are open-ended, allowing for multiple problem-solving strategies. Therefore, the tasks in this collection rate high on cognitive demand when analyzed against the Task Analysis Guide (see Appendix E) developed by Stein, Smith, Henningsen, and Silver. In this guide, a task that rates high on cognitive demand requires complex and non-algorithmic thinking, exploration and in-depth understanding of key concepts, connections between mathematical ideas, and an analysis of constraints. Each of these characteristics were important considerations for the selection of the task for this study.

To select the actual problem that the teachers would implement for this study, I worked collaboratively with both teachers to review the collection of tasks and select a common task that fit within the scope and sequence of their curriculum, and which adhered to the qualities described above for a high level of cognitive demand. Since both teachers taught geometry, they selected a task from the MAP collection that focused on properties of polygons and finding the interior and exterior measure of angles of polygons. Both teachers had done some prior work with their students on polygons and felt the problem would be developmentally appropriate for their students. Teacher A planned to use the problem as scaffolding within a larger project on the properties of polygons, while Teacher B intended to use the problem within a traditional unit on polygons. The teachers were given a copy of the problem, along with an instructional guide for the problem that provided suggestions for implementation and common student errors (see Appendix E).
Data Collection

To better understand each teacher's beliefs about teaching and learning mathematics and their instructional practice and how that might influence their implementation of the problem, a pre-implementation interview was conducted prior to their use of the problem with their students (see Appendix A). In this interview, the teachers described their teaching style, learning objectives for their students, instructional strategies, assessment strategies, plans for implementing the problem, anticipated student actions and challenges, and background information about themselves and their students. Data from the pre-interviews for each teacher were then analyzed and compared to determine specific similarities or differences in the teachers' beliefs and practices, and to explore how those similarities or differences might have impacted their implementation of the problem. For example, the teachers each described how they might respond if students struggled with the problem. Their descriptions were compared to each other to determine any differences in instructional approaches. In addition, the descriptions were compared to the notes from each of the teachers' actual implementation of the problem to assess whether their intentions matched their actions in the classroom.

Following the pre-implementation interview, the teachers each implemented the task in their classroom at approximately the same time in the school year. The lesson was observed and recorded for the purpose of analyzing the video for identification of specific teacher actions and comparison of the strategies used by each of the teachers, and how those actions led to either the maintenance or decline of the cognitive demand of the task. A framework for analysis was built using the top five factors found by Stein et al. (1996) for maintaining a high level of cognitive demand, listed below:
• Building on prior knowledge

• Scaffolding, including the types of questions asked by the teacher, the amount of inquiry students engage in throughout the lesson, and the type of discourse that occurs in the classroom

• Providing appropriate amount of time

• Modeling high level performance

• Sustained pressure for explanation and meaning

Each of these factors provides support for student learning without reducing the cognitive demand of the task. For example when a teacher uses questioning strategies to scaffold the learning they are pushing students to make connections and access prior knowledge rather than explaining how to solve the problem. In addition, engaging students in inquiry and providing an appropriate amount of time for that inquiry to occur allows students to engage with the ideas at a deeper level than mimicking the work of the teacher.

These factors were used to analyze the classroom instruction in order to identify similarities and differences between the teachers and to assess the level of cognitive demand in each classroom over the course of the lesson. Video from each of the teachers’ classroom was reviewed and their actions were categorized according to the factors listed above. For example, if a teacher referenced a prior lesson or prior concept that could be used in solving the problem, that action was categorized under Building Prior Knowledge. A detailed description of the teacher’s action was recorded for future comparison and analysis. Similarly, the types of questions used by the teacher were recorded in order to gauge the level of scaffolding provided for students by the teacher.
In addition, the video of each teacher’s class was broken down by different types of activities and timed in order to determine the percent of class time spent on each type of activity. The types of activities included the introduction of the task, independent work time, and guided instruction. For example, if students were given time to work on the task largely unassisted from the teacher, that class time was categorized as independent work time. In comparison, if the teacher led a classroom discussion or provided some level of instruction to the students, that class time was categorized as guided instruction. Any time spent introducing or clarifying the problem and associated expectations for student work and outcomes was categorized as introduction to the task. By calculating the percent of total class time spent on each type of activity and determining when those activities occurred in the lesson, I was able to find patterns in the teachers’ instructional practices, identify any key differences between the teachers, and gain a sense of when the cognitive demand was maintained or lowered during the lesson.

After completion of the lesson, a post-implementation interview was conducted to capture the teachers’ reflections about the problem, their implementation of the problem, and the degree to which the level of cognitive demand of the problem was maintained. These results were used to further analyze the implementation of the problem by aligning the teachers’ statements about the lesson with actual events of the lesson. For example if a teacher stated that they used discourse during the implementation of the problem, the video was analyzed to determine the degree to which discourse was actually used during the lesson. A comparison of the teacher responses and the corresponding analysis was then conducted to identify potential implications of the findings.
Upon reviewing the data, more information was needed about each of the teachers’ beliefs about teaching and learning of mathematics. A survey was developed using Swan’s (2005, p. 5) model of Transmission View versus Connected View of mathematics, as shown in Figure 2.

![Figure 2. Views of the nature of mathematics, learning mathematics, and teaching mathematics.](image)

Findings from this survey were combined with the data from the pre- and post-interviews in order to explore how the teachers’ beliefs and attitudes about the nature of mathematics, their instructional practices, and how students’ best learn math may have impacted their implementation of the problem. For example, if a teacher was more
aligned with the Transmission View, the data from their classroom was analyzed to find examples of instructional strategies that fit within the context of that view. Similarly, their survey results were compared with their descriptions and reflections about their classroom practices and objectives in both the pre- and post-interview to gain a deeper understanding of each teacher and how they approach teaching and learning in their classroom and the resulting effects of that approach.

The data from the analytical framework based on the work of Stein et al. (1996) on cognitive demand and the beliefs survey based on Swan’s views of teaching and learning in mathematics will be combined and used to construct two case studies of the teacher’s implementation of the problem. These case studies will provide a detailed picture of each teacher’s implementation of the problem and whether the level of cognitive demand was maintained or lowered throughout the lesson. Discussion about the results of the data and possible implications are provided in the following chapter.
CHAPTER 4
DATA ANALYSIS

This study explores how two math teachers implemented the same mathematical task with their students and the instructional decisions and actions that led to the maintenance or decline of the level of cognitive demand. This chapter presents the case of each teacher as a means for comparing both their implementation of the task and the variety of factors that may have influenced that implementation, including their prior experiences and training, pedagogical beliefs and practices, and goals for student learning.

The Case of Teacher A

Teacher Background

During this study, Teacher A was in his second year of teaching. He has a degree in biochemistry and holds a teaching credential in biology, chemistry, and foundational math. During his first year of teaching, he taught biology and a supplemental math class. The supplemental math class was a class provided basic skill remediation for students that were struggling with mathematics, as determined by standardized test scores. At the time of this study, he taught geometry as part of an integrated course in which he team taught with a digital media teacher. There were 54 students in the class.

Teacher A described his instructional practice as primarily project-based, in which students work in teams on complex projects that last three to four weeks in length. His projects were integrated and covered both math and digital media content, although he was only responsible for the math portion of the project. He described that he had been exploring the use of smaller problems or tasks as a means for covering the topics or
standards that did not fit within a larger project or as scaffolding within a project. However, he also stated that he tends to fall back on traditional instructional practices when it comes to helping students to learn the mathematical content and that the projects were primarily used to increase student engagement. Although the teacher had access to district approved textbooks, he felt that he generally used the textbooks as a resource for students to use when working on projects. For example, he explained that students could use the textbooks to look up definitions or formulas if needed. Additionally, all students had a laptop that they could use and the teacher cited the laptops as the primary resource for students. When asked how the students typically use the laptops, he explained that they use the laptop to look up information, share their work, use various software programs, and create presentations.

He described his own experiences with learning mathematics as fairly traditional in nature; however, he attended a high school that used project-based learning in other subject areas and felt as though the inquiry-based nature of projects would be successful in mathematics. He stated his overall learning goals for his students was to develop an understanding of the content so that they could be successful on the state standardized assessment, as well as to learn other skills such as collaboration, communication, and critical thinking. He also stated that he wants his students to become more self-directed learners and that he hopes the use of problems and projects can help them to develop that skill.

When asked to describe his ideal class, Teacher A stated that he would like for there to be clear routines and structures in place that would allow students to get to work and complete the task without being off task. He cited behavior challenges as the
primary obstacle for being able to engage students in inquiry-based learning. Although he remained committed to using a project- or problem-based approach, he felt as though he had not experienced much success with those approaches up to that point. When asked how he supports struggling students he referenced working one-on-one with the student or using small group workshops in which he pulls a small group of students together in order to provide more personalized support. In regards to assessment practices, Teacher A mentioned both formative and summative assessments, including journal writing, quizzes, tests, and presentations. He also stated that he relies heavily on his interactions with students during class to assess their knowledge and skills.

Plans for Implementing the Problem

When asked about his plans for implementing the problem, the teacher referenced what topics he would cover prior to introducing the problem to students, but did not discuss any specific planning in regards to the problem itself. His stated goals for student learning during implementation of the problem were that students would understand supplementary and complementary angles and interior and exterior angles of polygons. He described student success as students making progress on the task and being able to work effectively in their groups.

Analysis of Implementation

In order to deconstruct the teacher’s implementation of the task, the video of the lesson, which lasted one class period, was analyzed to find evidence of instructional strategies aligned with the leading factors for maintaining the level of cognitive demand, including building on prior knowledge, scaffolding, providing an appropriate amount of time, modeling high level performance, and sustaining pressure for explanation and
meaning. By applying this framework to the video of the teacher's implementation of the problem, I was able to identify the strategies that maintained the level of cognitive demand. In addition, by breaking the video down by type of activity, I was able to determine the percent of class time that those high cognitive demand strategies were used and identify patterns in the teachers' instructional decisions. In total 40 minutes of class time were recorded and analyzed using the framework described above.

**Building on prior knowledge.** Although the task required skills and knowledge from previous lessons, Teacher A made no reference to these concepts when introducing the task to the students. Analysis of the video of the lesson revealed that only 2% of the total class time was spent introducing the task, and that time was largely focused on giving instructions for work. For example, the teacher explained that the students should work in their teams to solve the problem and that they needed to answer all of the questions to the best of their ability. Once instructions for the work time were given, the students were allowed to begin working on the problem. As students began working on the problem, the teacher walked around and answered students' questions. During this time, there was evidence of building on prior knowledge as the teacher would often prompt students to think about prior learning that might be helpful in solving the problem. He used this strategy when students were struggling or asked him a question about the problem. An example of a prompt from the teacher that pushed students to think about prior knowledge is provided below:

Student: How do I find this angle?
Teacher: What do you know about the polygon?
Student: It has 5 sides?
Teacher: What else? What information did they give you about the problem?

Student: That it is regular?

Teacher: What do you know about regular polygons?

Student: They have equal sides and angles

Teacher: How can that help you to find the measure of one angle?

In this exchange, the teacher was helping students to access prior knowledge about the properties of regular polygons. To solve the problem, the students needed to apply that knowledge in order find a specific angle measure. During this first part of the class, the teacher continued helping individual students or small groups of students using similar prompts. Most of the students in the class seemed to be struggling with the problem and waiting for the teachers' assistance.

After approximately twelve percent of the class time, the teacher pulled the class together for a whole class discussion. During that discussion, there was evidence of the teacher using strategies intended to build on their prior knowledge. Similar to the exchange between the teacher and student above, the teacher engaged the class in a discussion around how to use the properties of regular polygons to find the measure of an interior angle of the polygon by prompting the students to consider what they know about regular polygons and how they might use that information:

Teacher: What information is given in the problem?

Student 1: It’s a regular polygon

Teacher: And what does regular mean?

Student 1: All sides are equal
Student 2: And angles
Teacher: So if all of the angle measures are the same, what would we need to know to find one angle measure?
Teacher: If we knew the sum of the interior angles, could we find one angle?
Student 3: We would just divide by 5, right?
Teacher: Yes. So do we know how to find the sum of the interior angles? How do we do that?

In this class discussion, the teacher was trying to help the students to elicit prior knowledge that they could apply in this problem. Without this prior knowledge, students would not be able to solve the problem. Prior to helping students to access this prior knowledge, many students were struggling with how to begin the problem and were often off task and not working on the problem.

**Scaffolding.** The instructional strategies that help to maintain the level of cognitive demand through scaffolding of the learning for students include questioning, engaging the students in inquiry, and facilitating discourse (Stein et al., 1996). Evidence of all three of these strategies was found in the actions of Teacher A throughout the implementation of the problem to some degree. Upon examination of the types of questions that Teacher A asked the students, it was evident that the questions were generally open in nature, as they required students to think and respond in a reflective way. In addition, the questions generally pushed students to make connections between the information given in the problem and previously learned concepts. For example, the teacher routinely asked questions such as “what do you know already that can help you solve the problem?” or “can you explain how you found that angle measure?” These
types of questions require students to critically examine the problem and make use of relevant knowledge and experiences, as well as justify and communicate their reasoning. Analysis of the video showed that Teacher A used these types of questions in both one-on-one interactions with students, as well as during whole class or small group instruction.

During the whole class or small group instruction, which composed approximately 76% of the class time, the teacher used questioning strategies similar to those that have been found to engage students in both inquiry and discourse. Chapin, O’Connor, & Canavan Anderson (2009) found that specific questions, or “talk moves” (pp. 12-18), can deepen students’ understanding of the mathematical concepts as they consider and expand on the ideas of others. This collective knowledge construction helps all students to go deeper into the concepts. Teacher A used some of the talk moves outlined by Chapin et al. (2009) that have been found to push students into inquiry and discourse. For example, during a small group workshop the teacher engaged in the following dialogue with students:

Student 1: I don’t get how to find angle AED
Teacher: What do we know?
Student 1: There are 540 degrees in a pentagon
Teacher: What else do we know?
Student 2: It’s a regular pentagon
Teacher: What does it mean to be regular?
Student 3: We know all the angles are the same so we just divide 540 by 5
Teacher: Why do we divide by 5?
Student 3: There are 5 angles and they all have to be the same. So you just divide.

Teacher: What did you get when you divided?

Student 1: 182

Teacher: Did anyone do it differently?

This questioning pattern allows students to build off of the ideas of others and to pull out and make use of given information. In addition, it asks students to explain their reasoning, which helps them to clarify their ideas for themselves and others. However, this questioning pattern was not consistent throughout the implementation of the problem. During the whole class discussion, he had one student come to the board to share his answer, but did not require that student to explain his reasoning. When another student offered a different answer, limited discourse was used to explore the discrepancy between the answers. Rather, the teacher suggested that they use the internet to look up the formula, which diminished the level of inquiry for students. In addition, at another point in the whole class discussion, a student stated that she did not understand the solution presented by another student. Rather than allowing the presenting student or other students to respond, the teacher answered her question directly. This action stopped any discourse from occurring with the class.

Providing an appropriate amount of time. Knowing how much time to give students to work on a problem or task is a significant challenge for teachers, especially in a class composed of students with a wide range of skills and abilities. Stein et al. (1996) found that teachers often provide too little or too much time for students, resulting in a lower level of cognitive demand. Too little time prohibits students from engaging in
inquiry and discourse at a meaningful level and too much time typically results in off-task
behavior and lack of focus on the mathematics. Both actions diminish the students’
opportunity to understand the content at a deep level. In the case of Teacher A, his class
time was distributed as shown in Table 1.

Table 1.

*Type of Activity and Percent of Time – Teacher A*

<table>
<thead>
<tr>
<th>Type of Activity</th>
<th>Percent of Time (minutes)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction to the task</td>
<td>2% (1 minute)</td>
</tr>
<tr>
<td>Independent Work Time</td>
<td>22% (9 minutes)</td>
</tr>
<tr>
<td>Guided Instruction</td>
<td>76% (30 minutes)</td>
</tr>
</tbody>
</table>

Although the teacher intended to allow students to work on the problem in their groups
for most of the class, the teacher intervened with this time and a majority of the class time
was actually spent in guided instruction, either as a whole class or a small group
workshop. The whole class discussion was nearly half of the total class time, while the
small group instruction was approximately one third of the class time. During these
times of guided instruction, the students asked questions, responded to questions from the
teacher, presented their answers to the problem or problem-solving strategies, and
discussed various ideas related to the problem. Within this time, the teacher would often
give students sub tasks to work on as a means for exploring certain aspects of the larger
problem. The amount of time students were given to work on these sub tasks ranged
from 30 seconds to five minutes. For example, when investigating how to determine the
sum of the interior angles of a polygon, the teacher led the students as follows:
Teacher: How many degrees are there in a triangle?

Students: 180 degrees

Teacher: How many degrees are there in a quadrilateral?

Students: 360 degrees

Teacher: Do you see a pattern? What do you think it would be for a polygon with 5 sides? Take a second and come up with a guess with your group.

The teacher then allowed approximately four minutes for students to talk in their groups and come up with an answer to his question. The students appeared to be largely off task during most of this time, as evidenced by the nature of their conversations and lack of attention to the problem. Once the teacher brought the class back together, he had a few students offer a potential answer. The students had arrived at different answers, so the teacher asked the class who they agreed with and why. Most students stated that they agreed with the first student, however they did not articulate why. The teacher then prompted the students to use their computers to look up the answer and gave them an additional three minutes for this task. Again, students appeared to be largely off task for most of this time suggesting that the time provided for students was too long and did not serve to maintain a high level of cognitive demand.

Towards the end of the class period, the teacher offered to run a small group workshop for any students that were still struggling with the problem. Students were allowed to self-select whether they attended the workshop. Twelve students attended the workshop while the rest of the class was given independent work time to complete the problem. During the workshop the teacher answered questions about the problem and engaged the students in discourse about the problem. This time was approximately one
third of the total class time. Thus there was a significant range in how much time individual students were allowed to work independently versus working in a more guided way with the teacher.

**Modeling high level performance.** Providing students with model of high level performance allows students to better understand the expectations for their work, as well as understand the complexity of the task. In addition, models can be used to further student thinking by asking them to deconstruct, analyze, or interpret the model. This pushes students to consider new ideas and potentially challenges some of their own ideas, which scaffolds their ability to operate at a high level of cognitive demand.

Very little evidence of modeling high level performance was found in the actions of Teacher A. Models were not presented at any point in the lesson, other than having a few students share out their solution and strategies during the whole class discussion. However, those samples did not represent high level performance because they primarily focused on their solution and not a detailed explanation of the concepts behind their work. In addition, the teacher did not discuss expectations for student work in either the introduction of the task or throughout the lesson, even though the teacher had identified in the pre-implementation interview that he

**Sustaining pressure for explanation and meaning.** A means for moving students to deeper learning throughout the problem-solving process is to require students to explain and justify their thought process and ideas. Communicating their ideas both in writing and orally helps them to make meaning of their work and to make connections between their ideas, the ideas of others, prior knowledge, and new information.
The degree to which Teacher A sustained pressure for explanation and meaning ranged significantly throughout the class time. In some instances the teacher used questioning techniques to have students explain their reasoning, however this was not consistent and often when students provided a response to a question the teacher did not require them to further explain their thinking or justify their strategy in any way. For example, in a one-on-one interaction with a student the teacher asked the student what they found, but did not ask them to explain how or to consider the significance or accuracy of their finding.

Student: I don’t understand how to find angle AEJ

Teacher: What have you found so far?

Student: This angle is 182 degrees

Teacher: What about this angle?

Student: That would be 182 degrees too

Teacher: Okay, so how many degrees total around point E? If we went all the way around it, how many degrees would there be?

Student: I don’t know

Teacher: Think about if we drew a circle all the way around the point. How many degrees are there in a circle?

Student: 360 degrees?

Teacher: Yes! Now can you use that to find the angle?

Student: Oh. Okay, I got it.

In this interaction, the teacher asked the student what they found, but he did not ask them to explain how they found the answer or why they thought their results were accurate,
which resulted in a missed opportunity to both assess student thinking as well as to help them make sense of what they know and how to apply that knowledge. In a similar interaction with another student, the student was struggling with the fact that the problem did not provide any given angle measures. The student asked the teacher the following:

Student: I don’t understand how to do this without any numbers

Teacher: Well, what do you know? What did they tell you about the polygon?

Student: There are four pentagons?

Teacher: Right. So you should be able to use what you know about pentagons to find the interior angles.

Student: Can I use my protractor to measure the angles? Is that okay?

Teacher: Sure, go ahead.

In this interaction, the student proposed a strategy and use of a tool that would not work given that the diagram was not drawn to scale. However, the teacher did not ask the student to consider and explain if that strategy would be valid and yield accurate results.

During the whole class instruction, the teacher asked a student to present their work to the class. The student went to the front of the room and recorded their work on the board for other students to see. The student offered some explanation as he was writing, but the teacher did not ask the student to explain his work in further detail. Once the student was done, the teacher asked the class the following:

Teacher: Who agrees with [student 1]?

Teacher: Did anyone come up with a different answer?

Student 2: I got 450 degrees

Teacher: How did you get that?
Student 2: Well, there are 360 degrees in a quadrilateral and if divide that by four you get 90 degrees. So then I just added another 90 degrees and got 450.

Teacher: Okay. Who agrees with [student 1]? Who agrees with [student 2]?

Why don’t we look it up?

In this interaction, the teacher did not push either student to defend their thinking or to elaborate on their ideas. By presenting two different solutions without an opportunity for students to make sense of the problem-solving strategy, students may become confused about the key ideas and where the errors in thinking occurred. In particular, the Student 2 presented an opportunity to explore and eliminate a misconception that many students might have had. By having students look up the answer as a means for deciding which solution was correct, the ability to help students to develop a conceptual understanding was diminished.

Some evidence of pushing students to explain and make sense of their ideas was found in the small group workshop, in which a small group of students engaged in dialogue with the teacher about strategies for solving the problem. During this time, the teacher used more questioning strategies that required students to expand on their ideas and the ideas of other, as described above in the description of the scaffolding. However, these questioning strategies were fairly inconsistent and somewhat limited. In addition, there was no evidence of the teacher supporting the students in how to clearly explain their reasoning. As shown in the problem task (see Appendix E), the question prompts for the problem ask students to explain their reasoning and show their work. When asked by a student how they should explain their answer, the teacher told them to just show their work. The teacher also did not address the prompt to explain their reasoning with
the whole class or discuss expectations in this regard. As a result, few students provided explanations with their work or explanations were limited. Some students only provided the answer to the problem.

**Teacher Reflection and Beliefs**

During the post-implementation interview, the teacher was asked to reflect on the implementation of the problem and the degree to which the cognitive demand was maintained or lowered (see Appendix B). In particular, the teachers were asked to describe how the students responded to the problem, their actions during the lesson, any changes they made to their lesson plan, whether the students met the intended learning goals, and how similar or different the lesson was to their normal classroom instruction. The teachers were also given a follow-up survey to collect additional information about their beliefs around the nature of mathematics, learning mathematics, and teaching mathematics (see Appendix C). The results of that interview and survey are described below.

**Teacher reflections.** Upon reflection of the lesson and the subsequent student learning, Teacher A stated that he didn’t know what to expect going into the lesson, but that the students responded very well to the problem and that their engagement level was very high, especially when compared to previous lessons. When asked why student engagement was higher, the teacher identified that the problem was within the appropriate skill level of the students and that it was interesting for them to try to figure it out because there were many different ways that they could have approached the problem. He also liked that the problem was relatively simple in appearance, yet complex to solve in that it required students to piece together information and use their
problem-solving skills. In addition, he stated that the problem prompted students to ask the right questions, which allowed him to guide them through the problem-solving process. When asked specifically about challenges students faced and his resulting actions, he described that some students were distracted by the fact that the problem did not have any numbers and that he pushed them to think about what they did know in order to solve the problem. He cited this challenge as the reason for pulling the class together early on in the lesson in order to help them to use the given information to start the problem, stating that it seemed like a majority of the class was lost or struggling to get started. Rather than continue to address students one-on-one, he decided to address the class as a whole. His intention was to provide just enough guidance to let them go back to independent work time. This was a departure from his original lesson plan, in which he had planned to let students work on the problem on their own for most of the class period and only use whole class discussion at the end to have some students share out their solutions and strategies. Overall, he characterized his role and actions throughout the lesson as checking in with students and using guiding questions to help them when they were struggling.

When asked to reflect on the cognitive demand of the problem, the teacher rated it as having a medium level of demand due to the fact that the students had little prior exposure to the concepts used in the problem, but were able to figure it out. He did not identify specific characteristics of the problem that would classify it as a medium level problem. He felt like the cognitive demand was partially maintained and that the whole class discussion was more effective in getting students to think critically than when they were given independent work time. When probed about this aspect of the lesson and how
it maintained the level of cognitive demand, the teacher referenced using guiding questions to help the students to better understand the concepts and stated that guidance is what they needed in order to get to the next steps. The teacher also mentioned that if he were to do the problem again, he would build in more collaborative strategies to help them to work more effectively in their groups. He would also have more sharing out and discussion throughout the lesson.

In regards to the learning goals, the teacher felt as though the goals were partially met and that they would need to revisit the concepts again at a later time, although he wasn't sure how he would do this. When asked how he knew if the learning goals were met, he said that when he was talking to students they seemed to be getting it and that he relied on his interactions with students to assess what they knew, especially during the whole class discussion. He planned to have the students use what they learned from the lesson in the project they were working on and would use small group workshops if students needed more help. He did not plan on using the student work in any significant way, other than quickly looking through their responses to determine how much they understood.

Overall, the teacher rated the lesson as very effective and stated that he would like to use similar problems in the future. He was surprised at how well the students responded to the problem, although he was unclear about what in particular made the problem so engaging.

Teacher beliefs. Upon analysis of the survey on teacher beliefs around mathematics, Teacher A had mixed results. While his view on the nature of mathematics leaned more towards the Connected View where math is seen as an interconnected body
of ideas and reasoning process, his views on student learning fell more on the Transmission View. In the Transmission View, learning is seen as a relatively individual process based on listening, watching and imitating. His beliefs about the teaching of mathematics were more aligned with the Connected View that focuses on the use of dialogue between student and teacher as a means for sense making. In the survey, the teachers rated their beliefs on a scale of 1 to 4, in which a rating of 1 indicated a strong alignment with the Transmission View and a rating of 4 indicated a strong alignment with the Connected View. The responses of Teacher A are shown in Table 2.

Table 2.

<table>
<thead>
<tr>
<th>Teacher Response</th>
<th>Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nature of Mathematics</td>
<td>3 (moderately aligned with the Connected View)</td>
</tr>
<tr>
<td>Learning Mathematics</td>
<td>2 (moderately aligned with the Transmission View)</td>
</tr>
<tr>
<td>Teaching Mathematics</td>
<td>3 (moderately aligned with the Connected View)</td>
</tr>
</tbody>
</table>

Although mixed, his results fell more towards the middle in all three areas as opposed to feeling strongly one way or another. When comparing his beliefs to his actions during the lesson and his reflections on the lesson, there is evidence that matches both views. For example, the teacher stated that he felt as though the whole class discussion was the most effective part of the lesson. During this time, he used guiding questions and discussion to help students to understand the content. While this would suggest alignment with the Connected View, analysis of the types of questions used showed that questions were more leading than open-ended which might fall more towards the Transmission View.
For example, the teacher often asked the students what information was given in the problem and then offered suggestions for how they might use that information versus asking an open-ended question that would prompt students to consider how to apply the given information themselves. In addition, while the teacher planned to let the students work collaboratively in their groups, he intervened relatively early on and provided more concrete guidance for the students.

His actions during the lesson also aligned with his own characterization of his teaching style from the pre-implementation interview in which he stated that he tends to fall back on more traditional strategies to help students understand the content. While not direct instruction, his actions during the whole class discussion were much more guided than inquiry based. This mixed approach reflects the state of his beliefs at this time of this study and demonstrate the complexity of the nature of teaching.

**The Case of Teacher B**

**Teacher Background**

During the time of this study Teacher B was in his fifth year of teaching. He is currently teaching exclusively mathematics courses, including a non-integrated Geometry course with 34 students in the class. This is his second year teaching Geometry. He has degrees in biblical studies and business administration and is certified to teach secondary mathematics. When asked why he became a math teacher he explained that he always felt like he was good at math and found it interesting. Thus when he decided to change careers and become a teacher, he opted to pursue a credential in mathematics.

He described his own schooling experience in math as traditional, in which the teacher would demonstrate a specific skill and then the students would practice that skill
independently. He felt that process was effective in helping him to learn the foundational skills of math and that routine practice is an essential part of learning mathematics. However, he also stated that he tries to incorporate more hands-on activities in his class, as student engagement and confidence with math is an important goal for his teaching. He is open to the use of problem-based learning in mathematics and experienced some success with the use of problems prior to this study. He identified the development of quality problems as the primary challenge to using a problem-based approach in his classroom. As such, he primarily uses the textbook to guide his curriculum and described his teaching style as more didactic in nature. However, he emphasized that he tries to build in collaborative activities for students and values collaboration and discussion among students.

When describing his overall goals for student learning in his class, Teacher B stated that he wants students to understand measurement and space and how to apply those concepts in various contexts. In addition, he wanted to help students to develop the ability to look at a scenario and use prior knowledge to establish a way to approach the problem and be able to analyze their answer to determine whether or not it makes sense. He cited communication as another key skill that he would like students to develop in his class and wanted to try to emphasize this skill more with his students on a regular basis by requiring them to provide written explanations of their work. Threaded throughout everything he said, it was also clear that he wanted to instill in his students a desire to learn and explore new ideas.

When asked to describe his ideal class, he described students work in groups, helping each other, explaining their thinking, showing their work, and persevering
through problems. He described his teacher actions as using Socratic questions to engage students in the process of critical thinking and discovery, as well as pulling out common threads for discussion with the whole class. In regards to strategies for differentiating instruction for students, he described using carefully selected mixed ability groups to allow students to learn from their peers, using questioning strategies to help students break the problem down and identify known information, using teacher assistants to provide peer tutoring, using models and diagrams, and having multiple students share out during class discussions so that students could hear multiple ideas and strategies. He also described his assessment strategies as consisting of regular quizzes, weekly write-ups of the problems they have solved during the week, and summative tests at the end of each unit.

**Plans for Implementing the Problem**

Teacher B planned to give the problem within a unit on polygons. Prior to implementing the problem the students had studied concepts that they could apply to solving the problem. He planned to give the students the problem handout and provide as much work time as they needed to complete the problem independently. He then planned on having students share out their work on the problem with the whole class and facilitates a discussion around the key ideas.

To assess their learning he planned on giving the students a formative assessment task, such as a quiz or follow-up problem and planned to rely heavily on discussions and sharing of ideas throughout the lesson to gauge student learning. He planned to use the results from the problem to target specific weaknesses of the students by providing
additional problems or extensions to the problem. If a majority of the class did not do well on the problem, he would try to find a way to re-teach the concepts.

Analysis of Implementation

The implementation of the problem by Teacher B was analyzed using the same framework used in the case of Teacher A. This process included identifying evidence or a lack of evidence associated with the factors that have been shown to maintain a high level of cognitive demand. In addition, the lesson was broken down by type of activity in order to determine how much time was spent on various aspects of the lesson. Combined, these two analyses provide a detailed description of the teacher’s implementation of the problem in order to address the question as to whether the level of cognitive demand was maintained or lowered throughout the process. In total 26 minutes of class time were recorded and analyzed using the framework described above.

Building on prior knowledge. In his pre-implementation interview, Teacher B described that he intended to use the problem as a follow-up to previous lessons that covered many of the concepts needed to solve the problem. He hoped that students would use this prior knowledge in order to extend their thinking about angle relationships and derive an understanding of exterior angles of polygons. As such, when he introduced the problem to the students he spent a significant amount of time discussing some of the key ideas that they could use in solving the problem. In total, he spent about a quarter of the class time (6.5 minutes) setting up the task. During this time, specific attention was paid to making connections to previous lessons. For example, the teacher repeatedly referenced prior lessons and how they might use that information when solving the problem, as shown below:
Teacher: Last week we spent a lot of time learning about polygons and exploring how to find the sum of the interior angles. You might need to use that when solving this problem. You also know about different angle types, so I want you to think about how you can apply what you know about angle relationships.

He also encouraged the students to use their notes and referenced a particular lesson that they could look at for examples if needed. In addition, he mentioned that they could use their textbook and identified the specific sections of the book that might be helpful. Although he was pointing them to specific resources and content, he did not give details on how they should use that information, which left the problem-solving process open to the students.

Once he gave the problem to the students, he read over the problem with the class and asked them a series of questions before letting them start on the problem. For example, he asked the students the following:

Teacher: What kinds of polygons are we given?

Students: Regular

Teacher: What do we know about regular polygons?

Student 1: All the angles are equal

Teacher: That is important to remember as you begin solving the problem.

Teacher: What other information are we given?

By breaking the problem down in this way, the teacher helped the students to identify what they knew, both from prior knowledge and problem scenario, and what they needed to know in order to solve the problem. This helped the students to build on their prior knowledge and make connections between concepts. Since this was done in the
introduction of the problem, most students were able to get started working on the
problem right away once they were provided independent work time. Many students got
their notes out or used their text book to look back at previous sections that they covered.
Some were able to work on the problem without looking at either their notes or the
textbook and seemed to have a good sense of how to get started. If students were
struggling to begin the problem the teacher referred them back to the information
provided in the problem and used questioning strategies to help them access and make
use of prior knowledge about polygons and angles.

Scaffolding. The teacher regularly made use of questioning strategies as the
students worked on the problem. Stein et al. (1996) found questioning to be a key aspect
of scaffolding the learning for students in a way that maintains a high level of cognitive
demand. The questions used by Teacher B often pointed students back to critical
information given in the problem or to concepts from previous lessons that they could
apply to the problem. However, his questions tended to be very specific in nature and
could be considered more guiding questions versus the Socratic questioning technique he
referred to in his pre-implementation survey. For example, when a student was
struggling he asked the following:

Teacher: What do you know about the interior angles?
Student: They have to all be the same
Teacher: Do you know the sum of the interior angles?
Student: Um, no.
Teacher: How have we found the sum of the interior angles in the past? What
formula did we use?
Student: I can’t remember

Teacher: Let’s take a look at your notes. Can you find that formula we derived?

In this interaction, the teacher guided the student to use a specific formula rather than probing their thinking about what they had tried already or what ideas they might have about solving the problem. In addition, when the teacher was asked a direct question by a student about the accuracy of their work, they typically provided an explanation versus using questioning strategies to push the student thinking. So while there was evidence of the teacher using questioning throughout the lesson, the types of questions may not have evoked high levels of cognitive demand and were inconsistent based on the interaction with the student. Questioning strategies were primarily used when a student was struggling and infrequently used when a student asked the teacher for feedback on their work.

The ability to maintain a high level of cognitive demand through the use of questioning strategies has a direct connection to engaging students in inquiry. By asking students probing questions such as ‘why do you say that?’ or ‘how do you know?’ students engage in an investigation of the information and use high order thinking skills to make sense of that information. Since the questions used by Teacher B were generally lower level questions, it was less obvious how the teacher sustained inquiry throughout the process through his questioning strategies. However, the teacher did push students to try out various problem-solving methods and did not provide students with a specific strategy for navigating the problem. In addition, he pushed students to persevere and apply what they knew already in order to solve the problem.
Discourse also played a limited role in the implementation of the problem with Teacher B. When introducing the problem, he asked students to work on the problem on their own and without the assistance of their peers. He explained that he wanted to see what they knew individually. As such, in the beginning of the lesson the students worked in isolation on the problem and little discourse occurred from student to student. However, the teacher did engage in discourse with students one-on-one. In addition, as more and more students seemed to be getting stuck on the problem, he allowed the students to work together. This occurred approximately nearly half of the way through the lesson. Generally, the level of discourse among students consisted of sharing of ideas and explanations of their work. There was no evidence of the teacher facilitating academic discourse with the students as a whole. However, it is important to note that the instructional guide for the problem did not include discourse as part of the suggested lesson plan and as stated in his pre-implementation survey, the teacher intended to follow the instructional guide as much as possible.

**Providing an appropriate amount of time.** In his pre-implementation interview Teacher B indicated that he intended to provide students with the majority of the class time to work on the problem. Analysis of his implementation of the problem revealed that the class time was divided into two types of activities: introduction of the task and independent work time. A quarter of the class time was spent introducing the task and the remainder was independent work time, as shown in Table 3.
Table 3.

*Type of Activity and Percent of Time – Teacher B*

<table>
<thead>
<tr>
<th>Percent of Time (minutes)</th>
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<tbody>
<tr>
<td><strong>Introduction to the task</strong></td>
</tr>
<tr>
<td><strong>Independent Work Time</strong></td>
</tr>
<tr>
<td><strong>Guided Instruction</strong></td>
</tr>
</tbody>
</table>

During the independent work time, a majority of the students appeared focused on the problem and the teacher circulated around the room to support students as needed.

Although the independent work time was extensive, the teacher kept students motivated and monitored their frustration level by using questioning strategies and scaffolds, as mentioned in the previous section. If students appeared to be frustrated or losing focus, the teacher would redirect them to the known information, a resource, or help them to activate the prior knowledge. As such, the amount of time spent on independent work time felt appropriate for the task and allowed all students to complete most of the problem. In addition, the introduction of the task and the focus on prior knowledge and connections between concepts during that time provided a foundation for the students to work from during the independent work time. Only minor clarifications were needed throughout the remainder of the class time and students appeared equipped to tackle the problem and persevere through the problem-solving without any guided instruction with the class as a whole. However, the teacher did adjust his original plans midway through the class and allowed students to work together. In his pre-implementation interview and in his introduction of the task to the students he stated that
he wanted students to work alone. This adjustment did not appear to affect the cognitive
demand of the problem as students engaged in sharing strategies and ideas and used this
opportunity to extend their own thinking. The amount of time provided for this level of
interaction among students seemed adequate to maintain focus on the task.

**Modeling high level performance.** Analysis of the teacher’s implementation of
the problem revealed no evidence of modeling high level performance. In the
introduction, the teacher referenced previous assignments and discussed performance on
those assignments. He used those examples to outline expectations for high level
performance on the problem task, including how students should communicate their ideas
and findings in writing. For example, the teacher described how they should structure
their responses:

Teacher: When you write up your answers you need to provide an explanation of
your work that anyone can understand, even if they don’t know the math. You
need to explain your reasoning in a way that would make sense to others. So it
needs to be clear, organized, and easy to follow. Explain each step of the way.
We’ve practiced this on other assignments, but we need to improve how we
explain and justify our reasoning.

This emphasis on communication skills correlates with the teacher’s goals for student
learning, specifically, developing the ability to clearly communicate their ideas.
However, while the teacher discussed performance expectations and made reference to
previous activities or assignments, he did not provide a concrete model of what high
quality work looked like in regards to effective communication. For example, the teacher
did not have students look at specific examples of explanations and have them analyze or discuss those samples as models for their own work.

**Sustaining pressure for explanation and meaning.** As stated above, the teacher placed a large emphasis on having the students explain their work. This was evident in the introduction of the task, as well as throughout the lesson. The teacher continually reminded the students that they need to provide detailed written explanations and would often give them feedback on their explanations as they worked. For example, when working with a student the teacher provided the following feedback:

Teacher: You've found the angle measure, but now I want you to describe the process you used to find that angle.

Student: What should I say?

Teacher: Think back through the steps you used and explain those steps to me so that it is clear. What did you do first? What information did you use and why? Explain that in your answer.

This example is representative of many of the interactions the teacher had with students that focused on explanation of their work in their write-up of the solution. In addition, the teacher often asked the students to explain what they were thinking orally to him. This prompt for explanation was particularly evident when the student seemed to be struggling with a concept. For example, in a one-on-one interaction with a student the teacher asked the following:

Teacher: Can you tell me how you got this value?

Student: I divided by 5

Teacher: What did you divide by 5?
Student: The total. How many there are all together. Because there are 5 angles inside.

Teacher: Alright. So how many degrees total would there be around this point?

In this example, the teacher was helping the student to break down their thought process so that they could apply a similar process to another part of the problem. However, there were additional opportunities within this example that would have helped the students to make deeper meaning of the concepts involved in solving the problem. The teacher did not ask the student how they found the total of all of the angle measures within the polygon, which was a key concept of the task. Many students simply used a formula out of their textbook to find this value; therefore it was unclear as to whether they understood the concept or merely were adept at performing the arithmetic involved in the calculation. Pushing students to explain their reasoning and make meaning beyond just explaining the steps they used to solve the problem would have resulted in a higher level of cognitive demand and more connections between concepts, as described in the Mathematical Task Analysis Guide (see Appendix D). According to this guide, while the initial task did not prescribe a specific strategy or procedure for solving the problem, the teacher's setup of the problem directed the students to a procedure they could apply without helping them to make connections or deeper meaning to the concepts behind that procedure.

Teacher Reflection and Beliefs

After implementing the problem with the students, the teacher was interviewed to gain a sense of their own thoughts on the effectiveness of the lesson and the level of cognitive demand throughout the problem implementation. In particular, I was interested in how they viewed the successes or challenges of the lesson and how this experience
with using a high level task may have affected their beliefs about teaching and learning in mathematics. This was captured in both the post-implementation interview (see Appendix B) and the survey of teachers' beliefs (see Appendix C). The results are discussed below.

**Teacher reflections.** During the post-implementation interview the teacher reflected on how his implementation of the problem differed from his typical instructional strategies. He felt as though the lesson did not differ significantly except the degree to which he let the students work on the problem without his direct guidance. In the implementation of the problem he tried to let them struggle a bit with the problem before he intervened and tried to use Socratic questioning when he did intervene rather than explain to them how to solve the problem. In comparison, when he normally gives students a task or activity he identified that he may intervene too quickly and give the students a clue or ask a leading question in order to reduce the amount of anxiety that students may be feeling. He reflected that normally his approach might also be more didactic and cited that he struggles to find tasks that are both engaging and provide an appropriate level of challenge.

In regards to his instructional actions during the lesson, he identified that he changed his original lesson plan when he decided to allow the students to work together mid-way through the lesson. He felt as though allowing some collaboration and discussion to take place helped struggling students to access the problem and that discussion was useful for the learning process for all students. He described his role during the process as asking the students questions and keeping them motivated to
persevere through the problem-solving process. This description correlates with his observed actions during the lesson.

Overall, the teacher was very impressed with the quality of the work produced by the students. He was surprised that the problem resulted in the high level of their responses and attributed this outcome to the fact that they were able to apply and extend their prior knowledge. He was also surprised by the high level of engagement of the students given that the problem appeared to be relatively simple and did not have a real-world application. Upon reflection on the problem, he identified that the problem was more complex than he anticipated as it forced students to look at relationships and make connections. However, when asked to assess the cognitive demand of the problem the teacher rated it as a medium level problem due to the fact that he didn’t think it required a great deal of critical thinking.

Teacher beliefs. Although the teacher described his instructional practices as more didactic with some use of problems and hands-on activities, analysis of his responses on the survey of teacher beliefs revealed that his views on the teaching and learning of mathematics are strongly aligned with the Connected View of mathematics. In this view, mathematics is seen as an interconnected set of ideas, learning is a collaborative activity, and teaching is primarily comprised of helping students to make meaning and connections through the use of dialogue. As described in Chapter 3, the teachers self-assessed their beliefs about three areas of mathematics using a scale of 1-4. The lower end of the spectrum represented beliefs associated with the Transmission View while the higher end of the spectrum represented beliefs associated with the Connected View. The responses of Teacher B are shown in Table 4.
Table 4.

*Teacher Beliefs Survey Results – Teacher B*

<table>
<thead>
<tr>
<th>Teacher Response</th>
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</thead>
<tbody>
<tr>
<td>Nature of Mathematics</td>
</tr>
<tr>
<td>Learning Mathematics</td>
</tr>
<tr>
<td>Teaching Mathematics</td>
</tr>
</tbody>
</table>

When comparing his beliefs with his actions during the lesson and his reflections following the lesson there is a strong degree of alignment. For example, the teacher identified communication and collaboration as an important part of the learning process during both the pre- and post-implementation interview. He also used strategies during the implementation of the problem that helped students to make connections between concepts. These strategies included using building on prior knowledge during the introduction of the task, using questioning techniques throughout the process to support and deepen student learning, and emphasizing the importance of clear explanations of their work.

In the final chapter I will present my interpretations of these two case studies along with some recommendations for both continued work with the teachers and potential areas for future research. I will also discuss the implications on using a problem-based approach as a means for maintaining a high level of cognitive demand.
CHAPTER 5
CONCLUSION

This study set out to examine the instructional practices of two teachers implementing a problem-based approach and how their actions maintained or lowered the cognitive demand placed on the students. As found by Stein and Smith (1998), both the problem design and the actions of the teachers had an impact on the level of cognitive demand throughout the lesson. Through an examination of the implementation of a high-level demand problem, I hoped to explore how the teachers’ actions aligned with the factors that lead to higher levels of cognitive demand as found by Stein et al. (1996) and how their beliefs about teaching and learning mathematics might have influenced those actions.

Each teacher has their own unique set of prior experiences and beliefs about the nature of mathematics. This study provided an opportunity to investigate and compare the actions and beliefs of two teachers through detailed case studies. I have utilized these case studies to consider the use of a problem-based approach for maintaining a high level of cognitive demand. By observing each teacher implement the same problem with their students, I was able to identify specific differences in both their planning and implementation of the problem. These differences inform potential implications for support of not only these teachers, but math teachers at large that are interested in using a problem-based approach in their classroom. In addition, this chapter will discuss potential implications of the findings of each case study on future research.

Summary

Analysis of each case revealed that the teachers implemented the problem in slightly different ways with their students. Evidence of some of the factors for
maintaining a high level of cognitive demand was present in both teachers' implementation of the problem, although to varying degrees. Their goals for student learning also differed in that one teacher was more focused on student performance on standardized assessments while the other teacher was focused on students understanding general concepts and developing the ability to effectively communicate their ideas and apply the mathematical skills. In addition, the teachers' beliefs about mathematics differed, although they were not entirely in opposition with each other. Similarities did exist in their reflections on the implementation of the problem, particularly their views on student engagement and performance. A summary of the findings for each case are described below.

**Findings from the Case of Teacher A**

Teacher A primarily used a project-based approach in his classroom prior to implementation of the problem used in this study. This approach has many of the same tenets as a problem-based approach in that students are given an open-ended task and must use critical thinking skills to arrive at a solution. Inquiry, dialogue, and questioning strategies are some of the main instructional strategies utilized in an effective project-based setting. As outlined by Stein et al. (1996), these are also some of the factors found in a problem-based setting for maintaining a high level of cognitive demand. Given the teacher's prior experience with a project-based approach I was interested in how well this would translate to a problem-based approach.

Upon analysis of his lesson, Teacher A did not employ many strategies aligned with maintaining a high level of cognitive demand. Most notably, little evidence was found around helping students to make connections to prior knowledge, especially during
the introduction of the task. In addition, the teacher did not push them to explain their reasoning in their written responses, even though he stated written communication as a learning outcome for his class. Very little independent work time was provided for students to wrestle with the problem. However, given that there was evidence of students struggling to engage in the problem early on in the lesson, the teacher’s instincts to reconvene the class and lead them in a whole class discussion about some of the concepts required for the problem showed that he recognized that the students needed scaffolding to be able to engage in the problem. In addition, during that time, he facilitated student discourse about the key concepts and used questioning strategies to probe student thinking. He also used this approach when facilitating the small group workshop near the end of the lesson. While teachers typically do not have control over how much time they have with their students, they do have control over how they use the time they do have time they do have with them is allocated. Therefore, examination of the allocation of class time may reveal insights into the teacher’s beliefs and priorities for student learning.

When examining his responses on the survey, he placed his beliefs about learning mathematics closer to a Transmission View than Teacher B. This was of particular interest given that he described his primary instructional approach as project-based, which would be considered to be more aligned with a Connected View. However, in his pre-implementation survey he stated that although he knows there are many benefits to a more inquiry-based approach such as project- or problem-based learning, he has not experienced much success with those strategies in his own classroom. When looking at his description of his learning goals for students, his goals were also more focused on student performance on standardized tests, which primarily measures procedural
knowledge and skills, which correlates with lower level cognitive demand on the Mathematical Task Analysis Guide. The combination of his beliefs, goals for student learning, and lack of success with a similar instructional approach may provide insight into why the teacher struggled to maintain a high level of cognitive demand throughout the lesson. The teacher’s transition to more guided instruction mid-way through the lesson also suggests that his prior experiences and beliefs impacted his actions.

**Findings from the Case of Teacher B**

In contrast to Teacher A, Teacher B self reported using more traditional instructional strategies in his class during the pre-implementation interview. However, interestingly enough, his views of teaching and learning in mathematics were more aligned with the Connected View of mathematics when measured on the beliefs survey. During the implementation of the problem, his actions reflected many of the factors that have been shown to maintain a higher level of cognitive demand for students. Specifically, he started the lesson by building on students’ prior knowledge and spent a significant amount of time helping them to make connections to their previous work. This strategy proved to support the students’ ability to access the task, as students were able to begin working on the problem without much guidance from the teacher. The teacher also provided ample time for students to work on the problem. Too little time could have resulted in incomplete answers or rushed work. By providing an extended period of time, he was able to push students to explain and justify their work in their responses. Although the teacher only provided limited modeling and did not engage the students discourse, he did use questioning strategies to help students to make connections and build on their prior knowledge, especially when students were struggling and needed
more guidance. However, analysis of the questioning used with students also revealed some missed opportunities to push students to deeper levels of thinking, thus reducing the level of cognitive demand of the task.

In the post-implementation interview the teacher commented that he was surprised at the level of student engagement and the quality of their work. As such, he stated that he would like to implement similar tasks with his students in the future. Given that Teacher B described his instructional style as more didactic, his ability to implement the problem and maintain a high level of cognitive demand points to the importance of beliefs about teaching and learning. In addition, his goals for student learning supported the strategies he utilized throughout the lesson.

**Combined Analysis**

Analysis of the data showed that the teachers’ beliefs about teaching and learning of mathematics correlated with their actions during the lesson more so than their general teaching style. As described in the pre-implementation interviews, the teachers had different instructional styles, one of which aligned more with a problem-based approach. However, that alignment did not appear to aid the teacher in the implementation of the problem. The teacher with the more didactic instructional style demonstrated more evidence of maintaining a high level of demand throughout the lesson and vice versa. In contrast, examination of their beliefs and goals for student learning did reveal a level of alignment with their actions during the implementation of the lesson. The teacher that focused more on helping students to make connections and explain their thinking had higher levels of student engagement with the problem than the teacher who focused more on developing specific skills.
Both teachers missed opportunities to probe student thinking at a deep level, particularly during their use of questioning strategies. Questions tended to remain focused on helping students to make use of given information or build on their prior knowledge as opposed to requiring students to justify their ideas with evidence and make meaning of the concepts. In addition, neither teacher used modeling of high level performance in any significant way throughout the lesson. This lack of modeling was also a missed opportunity to push student thinking. Analysis of sample work can help students to make meaning and sort out how concepts are connected. In addition, it communicates to students what level of performance is expected of them. In his post-implementation interview, Teacher B made reference to using models in previous lessons to have students engage in error analysis, however, that was not present in his implementation of the problem used in this study.

Of particular interest was how the teachers assessed the cognitive demand of the problem. Both teachers rated the problem as a medium level problem and associated the level of demand with the difficulty of the problem or the degree to which the problem required students to apply the skills in a real-world context. While a problem may be difficult for students and require many steps, this does not ensure a high level of cognitive demand. Similarly, a problem that includes a real-world context also does not mean that the problem is cognitively demanding. As outlined in the Mathematical Task Analysis Guide, tasks with a high level of demand require complex and non-algorithmic thinking. In other words, there is no suggested strategy for solving the problem and students must make connections between concepts and ideas. In addition, students must access relevant knowledge and experiences and make appropriate use of them in working
through the task. Students must also analyze the constraints of the task and explore the nature of the mathematical relationships. All of these indicators of a high level problem were present in the task used in this study. In the interviews with the teachers, both teachers indicated that finding high quality problems was a barrier to utilizing a problem-based approach with their students. Thus their assessment of the level of the problem raises a question about their ability to select problems with a high level of demand. Neither teacher identified any of the characteristics of a high level problem as outlined in the Mathematical Task Analysis Guide when looking for problems to use with their students, even though both teachers had participated in professional development sessions that used that guide to examine and discuss sample problems.

The post-implementation interview revealed that the teachers did not make use of the student work produced during the lesson in any significant way. When asked whether the students met the learning goals for the lesson, both teachers felt they had met the goals but only cited evidence from their interactions with the students during the class time. When asked specifically how they would use the student work, the teachers indicated that they would look through the work to get a general sense of what the students learned or what they might need to follow-up on, but they did not plan to spend a great deal of time on analysis of the student work or provided feedback for the students. In my experiences working with a wide range of math teachers, this seems characteristic of how most teachers view student work. Many teachers regard student work as an end product, rather than an opportunity for further learning to take place. Nor do they use it to reflect on their instructional practices and guide instructional decisions moving forward.
Overall, both teachers exhibited some of the factors associated with maintaining a high level of cognitive demand. Each teacher excelled or struggled in different areas and their actions seemed to be influenced by their beliefs about how students learn math, as well some of their prior experiences. For example, even though Teacher B self-described his style as more didactic, he also valued collaboration and communication and had experience embedding this skills in his day-to-day lessons. While Teacher A typically used an approach that could be considered more aligned with a problem-based approach, his goals and beliefs about student learning did not support that approach and may have contributed to some of the challenges with that approach that he described in the pre-implementation interview. However, despite their different experiences with the implementation of the problem, at the end of the study both teachers remained open to the use of a problem-based approach with their students and expressed a desire for more support and access to quality problems in order to achieve higher levels of student thinking and understanding of the concepts.

**Implications**

Analysis of the teachers' actions throughout the implementation of the problem revealed that the teachers struggled with different aspects of maintaining the level of cognitive demand of the problem. This analysis and subsequent findings have implications for these specific teachers, for my work as an instructional coach and professional development of teachers, and for future research in this area. These implications are discussed below along with general reflections on this study.
Implications for the Teachers

In comparing the implementation of the problem by the two teachers, Teacher B seemed to be able to sustain the level of cognitive demand more than Teacher A, however he had specific challenges as well. This finding indicates that both teachers need additional support for using a problem-based approach in their classroom. In particular, the teachers need regular feedback on their instructional practices. Just as students need feedback and opportunities to reflect on their learning, teachers need to have opportunities to get timely feedback in order to reflect on and adjust their instructional practices. Too often, teachers work in isolation and have few opportunities to collaborate with their peers and discuss instructional strategies. In the case of these two teachers, particular focus should be placed on helping them to utilize effective scaffolding strategies that maintain a balance between supporting student learning and pushing them to think at deeper levels. Often, teachers provide too much guidance and end up reducing the level of demand placed on the student. Questioning strategies, opportunities to engage in discourse, and guided inquiry can provide a support structure that helps students to connect ideas, make meaning of the concepts, and correct any misconceptions. While both teachers demonstrated some facility with these approaches, professional development and feedback in this area could increase their ability to use these strategies more effectively.

In the case of Teacher A, coaching around how to build on the students’ prior knowledge may help him to set-up tasks in a way that make them more accessible to students. This in turn may result in a greater sense of success with the use of problems. In this study, he did not help students to make connections to prior lessons or concepts.
Many teachers that use a problem-based approach for the first time misinterpret the premise of not giving explanations before the problem as meaning that they should not provide support or instructions to students. As such, students typically struggle to begin the problem. In comparison, Teacher B made reference to previous skills and lessons but did not tell students how to solve the problem. This approach provided an appropriate amount of scaffolding for students to access the problem while maintaining a high level of cognitive demand.

Supports for Teacher B may focus on the use of modeling for high level performance, given that this was not evident in his implementation of the problem. In addition, more access to quality problems and helping the teacher to accurately assess the demand of problems would help him to gain more experience and confidence with a problem-based approach, as I believe his biggest hurdle was knowing what types of problems to give students and being able to develop problems of his own.

As revealed in the post-implementation interviews, neither teacher used the student work in any significant way. Both teachers stated that they would briefly look through the student work to gauge the level of student understanding, but relied more heavily on their interactions with the students during the class period in order to assess students’ knowledge and skills. More work needs to be done with these teachers around how to look at and use student work as a means for reflecting on their own instructional practices, as well as understanding the depth of knowledge that students were able to attain through the problem-solving process. Looking at student work can reveal a significant amount about misconceptions and errors that students are making, which can then inform the ongoing work of the teacher. Teachers need to reflect on both their
actions and student learning in order to be able to more effectively guide students toward the intended goals.

In future work with these teachers, it would also be helpful to take a deeper dive into the factors and experiences that have shaped their specific goals and beliefs for student learning. As shown in the analysis of the case of each teacher, their goals and beliefs were more aligned with their implementation of the problem than their prior teaching experiences. A better understanding of how they each view teaching and learning would assist me in working more collaboratively with these teachers by providing a frame of reference for how they approach their work. In other words, I can meet the teachers where they are and help them to negotiate any tensions between practices and beliefs.

**Implications for Instructional Coaching and Professional Development**

The analytical framework used in this study was very helpful in identifying the teachers’ strengths and challenges and subsequent coaching supports needed to help them improve in their practice. Often, when observing or working on curriculum and instruction with a teacher it is difficult to pinpoint specific things that can help them to provide a richer learning experience for their students. By examining their instructional practice for evidence of the top factors for maintaining a high level of cognitive demand, it was quickly apparent what the teachers did well, what they attempted to do but need more practice or support with, and what was largely absent from their instruction all together. This process then enabled me to quickly generate potential strategies for working with the teachers and target support for their areas of need.
The framework used in this study also led to the design of a coaching tool for observing teachers and providing feedback on their practice, as it pertains to the factors associated with maintaining a high level of cognitive demand (see Appendix F). I plan to use this tool with teachers that I coach in order to target specific areas of growth and support. In addition, the results of this study allowed me to see a potential progression of professional development sessions that could be built around the framework. For example, a strand of professional development sessions aligned to the factors for maintaining a high level of cognitive demand could be constructed and utilized to support teachers that are utilizing a problem-based approach. Collaborative activities around each of these factors would provide teachers with an opportunity to practice and reflect on the instructional actions that have been shown to have a large impact on student learning.

Since many teachers also struggle with understanding what defines a cognitively demanding problem, more training and support is needed in this area to help teachers construct or select high quality problems. Also, given that the problem is the first filter of cognitive demand, as shown in the Mathematical Tasks Framework, this is an essential element of any professional development around problem-based learning. Providing opportunities for collaboration among teachers would allow teachers to discuss problem design and come to greater clarity about what types of tasks will result in the intended learning outcomes. Opportunities for collaboration would also allow teachers to observe and discuss their implementation of high-level problems and develop a greater awareness as to how their actions support or hinder the quality of student learning. In my analysis of the teachers’ reflection of the lesson, I was also struck by how little clarity the teachers had about why the student engagement level was as high as it was and what factors
contributed to the level of student success. Teachers must have opportunities to explore these factors and engage in collaborative inquiry about student learning.

In general, many of the factors for maintaining a high level cognitive demand for students also apply to professional development for teachers. Helping teachers to build on their prior experiences, using discourse to have them explore instructional strategies, providing models of high quality problems, and sustaining pressure to have teachers explain their ideas for their lessons and make meaning between the problem design and student work would all contribute to providing high quality professional development experiences for teachers. Additionally, teachers need the time to develop problems and reflect on their actions and resulting student learning. Many teachers in the United States feel pressured to cover all of the topics in a given subject area in order to adequately prepare students for standardized tests. As a result, depth is often sacrificed for breadth.

**Implications for Future Research**

Upon reflecting on the results of this study, many questions still remain as to how much a teacher's beliefs about teaching and learning directly impact their actions in the classroom. While it is reasonable to believe there are connections between their beliefs and actions, additional research in this area would provide greater insight into how to support teachers with varying beliefs. By understanding how teachers' beliefs may help or hinder their ability to maintain a high level of cognitive demand, professional development can target specific areas for supporting teachers. In addition, we need to better understand how those beliefs are formed and the degree to which prior experiences shape those beliefs.
Additional research on the type of professional development that is effective in helping teachers to maintain a high level of cognitive demand would also contribute to more teachers being able to effectively implement a problem-based approach. As stated above, utilizing the analytical framework used in this study to construct a professional develop experience for teachers is one potential strategy for supporting teachers using a problem-based approach. Examining the use of that framework for professional development, as well as other professional development strategies focused on the use of problem-based learning is needed to better understand what teachers need in order to develop the skills to support student learning in this environment.

Final Reflections

While the cases used in this study provide an opportunity to explore the actions and beliefs that impact the level of cognitive demand of a problem, the scope of this study is confined to the implementation of one problem. By conducting an extended examination of teacher practices over time, additional findings may inform the work of teachers and those who support teachers. Given my work with math teachers in various contexts and prior experiences, a greater understanding of all the factors that influence their decisions and actions would allow me to provide more effective support. In addition, more insight into the use of a problem-based approach in mathematics to create engaging and rigorous learning opportunities for students would help others to see the value of that approach.

While many national organizations endorse the type of instructional model used during this study, teachers need well-designed, longitudinal professional development, greater access to high quality materials, and more opportunities to engage in collegial
conversations about their work in order to more effectively implement a problem-based approach. I believe this level of support is possible and hope this study is helpful in moving teachers down the path of creating meaningful learning experiences for their students.
APPENDICES
APPENDIX A
Pre Implementation Interview

Pedagogical Beliefs and Instructional Practices

1. What goals do you have for student learning in your class?
   - What would you like students to know and be able to do at the end of the school year? Why?
   - How will you know if students are meeting the goals? How will you measure the goals?

2. What does an ideal day look like in your classroom?
   - What are the student actions?
   - What are your teacher actions?
   - Think of a specific lesson that went particularly well. What made the lesson effective?

3. What instructional strategies do you use to differentiate for varying student abilities?
   - What do you do when a majority of the students in the class are struggling with a particular concept / skill?
   - What do you do when only a few students are struggling with a particular concept / skill, but the rest of the class is ready to move on?
   - How do you ensure that advanced students are challenged?

4. How do you assess student learning?
   - What kind of formative assessments do you routinely use? Why?
   - What kind of summative assessments do you routinely use? Why?
• How do you use the results of the assessments?

Implementation of Modules

5. How does the module fit within the scope and sequence of your curriculum?
   • Why did you select the module?
   • What learning goals do you have for students in the module?
   • What challenges do you anticipate students having during the module?
     How do you plan to support students with those challenges?

6. How will you plan for the implementation of the module?
   • What will happen before the module? And then?

7. What do you hope to see during the implementation of the module?
   • What will be the role of the students? What actions do you expect the
     students to make?
   • What will be your role during the module?

8. What will you look for during the module? How will you know if students
   understand the concepts?
   • What if a student/group finishes part of the module before the rest of the
     class but you aren’t sure they understand it?
   • What if a student/group can’t complete a part of the module in the
     allotted time?

9. What will follow-up of the module look like?
   • How will you use the students’ work?
   • How will you determine if the goals of the module have been met?
Background Information

10. How many total years have you been teaching?
   - How many years teaching high school math?
   - How many years teaching your current course?

11. What is your own educational background?
   - What did your high school math experience look like?

12. Describe the make-up of your class
   - Number of students?
   - Level of students?
   - Demographics of students?
   - Number of students with special needs? Types?

13. What curriculum and/or instructional materials do you use?
   - How well do the modules fit within or align with that curriculum?

14. What are some examples of mathematical tasks that you have given your students?
   - How would you rate the cognitive demand of those tasks using the Mathematical Tasks Framework (Stein, Smith, Henningsen, & Silver, 2000)?

15. What prior training / support have you had with the use of the modules?
   - What is your understanding of the instructional strategies used in the modules?
   - Why did you decide to implement the modules?
APPENDIX B

Post Implementation Interview

Planning

1. What were the student learning goals for this problem?
2. How did you plan for the implementation of the problem?
3. What changes to the instructional guide did you make and why?
4. What prior knowledge did you expect the students to utilize in this problem?
5. What challenges did you expect students to encounter and why?
6. How would you rate the cognitive demand of the problem and why?

Implementation

7. How did students respond to the problem?
   • What were the student actions?
   • Were there particular successes and challenges you noticed?
   • How would you rate the student engagement?
8. Describe your actions during the implementation of the problem?
   • How did you introduce the problem?
   • What did you do while students worked on the problem?
   • How did you wrap-up the problem?
9. Did you make any changes to your lesson plan midstream?
   • What prompted these changes?
   • How did you feel about the changes after the lesson?
10. Do you feel like you were able to maintain the level of cognitive demand of the problem? Why or why not? When and how did it increase / decrease / stay the same?

- Did your assessment of the level of cognitive demand change after implementing the problem? Why or why not?

11. Were you able to provide differentiation during the implementation of the problem? Can you describe a specific example?

**Reflection**

12. What would you do differently if you used the problem again? Why?

13. Did your students meet the intended learning goals? Why or why not? How do you know? What evidence did you use?

14. Other than the content goals, what other learning outcomes did the lesson target? How?

15. What’s next? How will you use the information about the students understanding?

16. What surprised you about the lesson?

17. How well did the problem align with your regular teaching strategies? Did anything feel awkward, foreign? Why?

18. Is there anything else that you feel would be valuable to share that has not been asked already?
APPENDIX C

Teacher Beliefs Survey

Reflect on your beliefs about the nature of mathematics, learning mathematics, and teaching mathematics and answer each of the questions below. Please note that neither descriptor is necessarily good or bad, but rather represents a spectrum of beliefs. If your beliefs fall within the statements provided, please indicate so on the scale.

1. Beliefs about Mathematics

Using the following scale, rate your beliefs about the nature of mathematics

| Mathematics is a given body of knowledge and set of procedures. | 1 | 2 | 3 | 4 | Mathematics is an interconnected body of ideas and reasoning processes. |

2. Beliefs about Learning

Using the following scale, rate your beliefs about learning mathematics

| Learning mathematics is an individual activity based on watching, listening and imitating until fluency is attained. | 1 | 2 | 3 | 4 | Learning mathematics is a collaborative activity in which learners are challenged and arrive at understanding through discussion. |

3. Beliefs about Teaching

Using the following scale, rate your beliefs about teaching mathematics

| Teaching mathematics is structuring a linear curriculum for learners, giving explanations before problems, checking that these have been understood through practice exercises and correcting misunderstandings. | 1 | 2 | 3 | 4 | Teaching mathematics is exploring meaning and connections through non-linear dialogue between teacher and learners, presenting problems before explanations, making misunderstandings explicit and learning from them. |
APPENDIX D

Task Analysis Guide

<table>
<thead>
<tr>
<th>Lower-Level Demands</th>
<th>Higher-Level Demands</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Memorization Tasks</strong></td>
<td><strong>Procedures With Connections Tasks</strong></td>
</tr>
<tr>
<td>• Involves either producing previously learned facts, rules, formulae, or definitions OR committing facts, rules, formulae, or definitions to memory.</td>
<td>• Focus students’ attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas.</td>
</tr>
<tr>
<td>• Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure.</td>
<td>• Suggest pathways to follow (explicitly or implicitly) that are broad, general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts.</td>
</tr>
<tr>
<td>• Are not ambiguous – such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated.</td>
<td>• Usually are represented in multiple ways (e.g., visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning.</td>
</tr>
<tr>
<td>• Have no connection to the concepts or meaning that underlie the facts, rules, formulae, or definitions being learned or reproduced.</td>
<td>• Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding.</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Procedures Without Connections Tasks</th>
<th>Doing Mathematics Tasks</th>
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<tbody>
<tr>
<td>• Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task.</td>
<td>• Requires complex and non-algorithmic thinking (i.e., there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example).</td>
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<tr>
<td>• Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it.</td>
<td>• Requires students to explore and to understand the nature of mathematical concepts, processes, or relationships.</td>
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<tr>
<td>• Have no connection to the concepts or meaning that underlie the procedure being used.</td>
<td>• Demands self-monitoring or self-regulation of one’s own cognitive processes.</td>
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<tr>
<td>• Are focused on producing correct answers rather than developing mathematical understanding.</td>
<td>• Requires students to access relevant knowledge and experiences and make</td>
</tr>
<tr>
<td>• Require no explanations, or explanations that focus solely on describing the procedure that was used.</td>
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</table>
appropriate use of them in working through the task.
• Requires students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions.
• Requires considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process required.
APPENDIX E

Problem Task

Four Pentagons

This diagram is made up of four regular pentagons that are all the same size.

1. Find the measure of angle AEJ.
   Show your calculations and explain your reasons.

2. Find the measure of angle EJF.
   Explain your reasons and show how you figured it out.

3. Find the measure of angle KJML.
   Explain how you figured it out.
### APPENDIX F

**Observation and Coaching Tool**

<table>
<thead>
<tr>
<th>Building on Prior Knowledge</th>
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<tbody>
<tr>
<td>Scaffolding</td>
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<td>• Questioning</td>
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<tr>
<td>• Discourse</td>
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<td>• Inquiry</td>
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<td>Appropriate Amount of Time</td>
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<td>Modeling High Level</td>
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<tr>
<td>Performance</td>
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<td>Sustaining Pressure for</td>
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<td>Explanation and Meaning</td>
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APPENDIX G

Rights of Human Subjects Protocol

Protocol Summary Sheet

<table>
<thead>
<tr>
<th>If requesting Exemption or Expedited Review, specify category (see Appendix B):</th>
<th>Title of Project: Instructional Strategies and Shifts in Teachers' Beliefs in a Problem-Based Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1 and A2 exemption</td>
<td></td>
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</tbody>
</table>

Brief description of purpose of project:

My research study will examine the shifts in both instructional strategies and teachers' beliefs when utilizing a problem-based approach in secondary mathematics. I will conduct a case study of two teachers' implementation of cognitively demanding problems and how the level of cognitive demand is maintained or lowered through their use of the problems.

<table>
<thead>
<tr>
<th>New project</th>
<th>Modification</th>
<th>Date Starting Interaction with Human Subjects: August 1, 2011</th>
<th>End Date: May 1, 2011</th>
<th>Funding Source (if any): N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sub-study</td>
<td>Previous study</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Subjects

Number: 2

Population: High School Math Teachers

Source/How contacted:

I discussed my project with a group of colleagues to determine if they would be interested in participating.

Instruments

Check all that apply: ☐ Tests ☐ Questionnaires ☐ Interview guides ☐ Other: N/A (unstructured interview)

Attachments: copy of each instrument used. if not yet developed, provide drafts, samples, and/or outlines

How administered:

☐ Telephone ☐ Mail or email ☐ In person Length and frequency of procedure: monthly up to one hour

Setting: Teacher’s School

Data

Check all that apply. Data will be recorded by:

☐ written notes ☐ audio tape ☐ video tape ☐ photography ☐ film ☐ other: Interview Notes

Data will include:

☐ information which can identify the subject (e.g., name, social security number, other unique identifier) specify:

For items checked above, circle box of those related to data that will be reported

Data will be used for:

☐ publication ☐ evaluation ☐ needs assessment ☐ thesis ☐ other

Informed Consent

☐ written (attach copy of consent form; see attached sample and checklist) ☐ oral (attach text of statement and request for waiver of written informed consent; see Appendix A)

This project:

☐ is exempt under category A-

☐ is eligible for expedited review under category B-

☐ requires CRHS review

Human Subjects Administrator Date

Chair, IRB Date

Comments:

This space for IRB use only
References


